Measuring the Angular Dependence of Cosmic Ray Muons

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In this paper, we measure the angular dependence of the cosmic ray muon flux using the cosmic watch muon detector. We show that our results are consistent with a $\cos^2\theta$ law for the flux, where θ is the polar angle with respect toe the vertical.

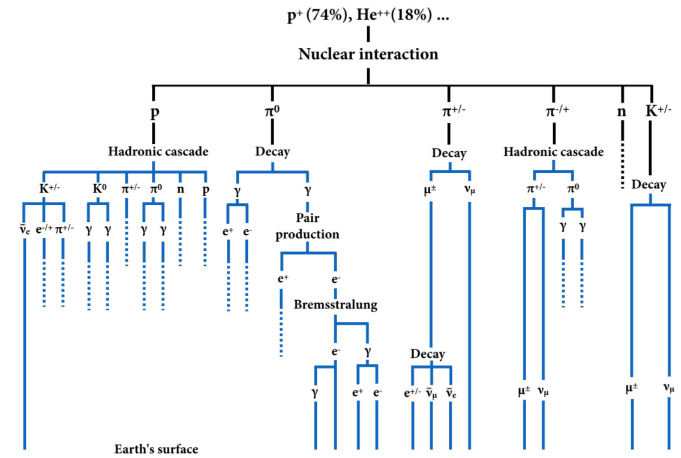

FIG. 1. Schematic representation of various interactions and decays that result in cosmic rays. Dotted lines represent particles that generally don't make it to the Earth's surface. Taken from [1].

I. MOTIVATION

Cosmic rays are high energy particles that originate from outer space that interact with Earth's atmosphere and generate a shower of other particles that strike the Earth's surface (which we also refer to as cosmic rays). These particles can come from a variety of sources, including the sun, distant stars, and even black holes. A schematic of the most common interactions in creating cosmic rays is shown in Figure 1.

The most common interaction with the atmosphere is a proton colliding with an air molecule, which leads to a decay that includes charged pions, which then decay into charged muons. These charged muons are the most numerous of the charged cosmic ray particles that make it to the Earth's surface, and are thus the easiest cosmic rays to measure.

We are interested in measuring the angular dependence of the flux of cosmic ray muons, which could give us insight into the secondary cosmic ray production process (the various interactions and decays that happen in the Earth's atmosphere).

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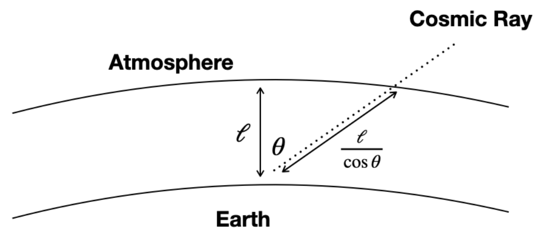

FIG. 2. Schematic representation of longer path length for cosmic rays entering at an angle

II. THEORETICAL BACKGROUND

We first define what we mean by the angularly dependent muon flux. Concretely, we are interested in $I(\theta, \phi)$, which is defined as follows. Given a plane of area A with normal in the (θ, ϕ) direction (here θ is the polar angle with $\theta = 0$ being vertical, and ϕ is the aziumthal angle), the number of muons striking it orthogonally within a solid angle $d\Omega$ in time t is given by

$$I(\theta, \phi) t A d\Omega.$$

In particular, note that $I(\theta, \phi)$ is measured in number of muons per unit area per unit second per unit solid angle.

Heuristically, we expect $I(\theta, \phi)$ to be decreasing as θ increases from 0 to $\pi/2$, since the path length traveled by the cosmic ray is roughly proportional to $1/\cos\theta$, see Figure 2. The reason for this is since the longer the path length of the muon in the atmosphere, the more energy it is expected to lose, and the higher probability that it decays before striking the surface of the Ea

Several large experimental groups have measured the dependence to be $I(\theta, \phi) \sim \cos^2\theta$, see for example [2]. Our goal is to observe the angular dependence of the cosmic ray flux and compare to this $\cos^2\theta$ law.

III. EXPERIMENTAL APPARATUS AND SETUP

III.1. Cosmic Watch Detector

The cosmic watch is a portable detector that measures cosmic ray muons through scintillation. A scintillator is

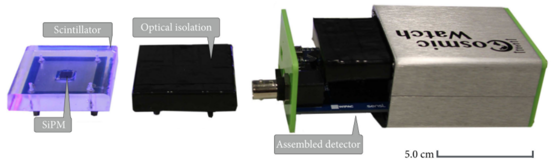

FIG. 3. Cosmic watch components. Taken from [1].

a material which may be excited by energetic charged particles (usually through Coulomb interactions) and re-emits some fraction of that energy as electromagnetic radiation. Often scintillators are made out of organic plastics.

The primary component of the cosmic watch detector is a 5 cm × 5 cm × 1 cm block of scintillator plastic. The scintillator is housed in an optically sealed case, and is connected to a photomultiplier, which sends a digital signal whenever an event occurs in the scintillator. See Figure 3 for a depiction of the components of the detector.

Another useful feature is the ability to identify to coincidences. The detectors have ethernet ports, and if they are connected with an ethernet cable, the display on the detectors only shows coincidence counts. This feature will be of paramount importance in our experimental design.

III.2. Experiment Design

The basic ideas is to place two cosmic watch detectors in parallel with their large sides facing each other. In particular, both detectors are secured to a wooden plank, exactly in the same orientation, and separated 35 cm. See figure 4 for an image of this setup.

FIG. 4. Image of detectors secured onto wooden plank and connected by ethernet for coincidence counting.

We then place the setup against the wall at various angles, and let the detectors record for some number of hours, and record the number of counts registered and the total runtime. We make sure to use the same wall each time, which ensures that all measurements have a consistent ϕ. This is because we don't have a good understanding of the ϕ dependence of I, besides the fact that it is generally slowly varying.

The idea here is that a coincidence count registering in both detectors usually comes from a muon traveling on path that passes through both detectors, meaning it is generally traveling with polar angle θ, if the detector is set up with angle θ with respect to the vertical.

III.3. Theoretical Prediction

In order to come up with a prediction based on the $I(\theta, \phi) \sim \cos^2\theta$ law, we need to analyze the various angles at which a cosmic ray can hit both detectors. Indeed, suppose let $\ell = 5$ cm be the side-length of the scintillator, and let $h = 35$ cm be the distance between the scintillators. Let θ' be the angle with respect to the normal of the scintillators. Note that cosmic rays with

$$-\sin^{-1}\left(\frac{\ell\sqrt{2}}{h}\right) \leq \theta' \leq \sin^{-1}\left(\frac{\ell\sqrt{2}}{h}\right)$$

can theoretically strike both detectors, and thus register a coincidence count.

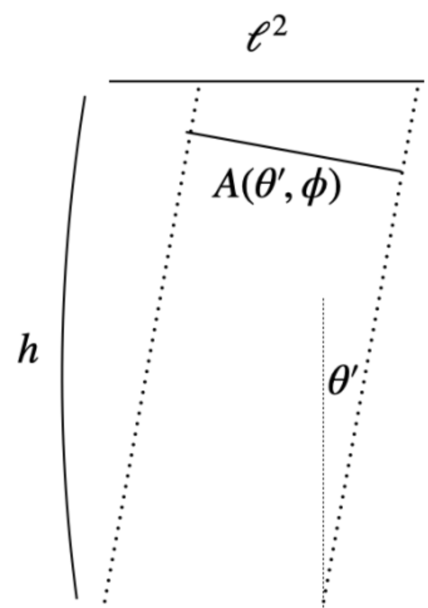

FIG. 5. Depiction of geometry of cosmic rays striking both detectors.

In particular, we need to find $A(\theta', \phi)$, which is the effective area of cosmic rays traveling with direction (θ', ϕ). See figure 5 for a schematic depiction of the geometry.

Note that

$$\frac{A(\theta', \phi)}{\cos\theta'} = g(h\sin\theta'\cos\phi, h\sin\theta'\sin\phi) =: f(\theta', \phi)$$

where $g(x, y)$ is the overlap between two axis aligned $\ell \times \ell$ squares with centers shifted by the vector (x, y). Thus, we find that the number of cosmic rays per unit time passing through both scintillators per unit time is given by

$$\int_0^{\pi/2} d(\cos\theta') \int_0^{2\pi} d\phi \, \cos\theta' f(\theta', \phi) I(\theta' + \theta, \phi).$$

Accounting for incidental coincidences not due to cosmic rays truly passing through both, we predict that the measured count rate $R(\theta)$ has the form

$$R(\theta) = \alpha J(\theta) + J_b, \tag{1}$$

where

$$J(\theta) = \int_0^{\pi/2} d(\cos\theta') \int_0^{2\pi} d\phi \, \cos\theta' \gamma(\theta', \phi) \cos^2(\theta + \theta'),$$

where $\gamma(\theta, \phi) = g(h \sin\theta' \cos\phi, h \sin\theta' \sin\phi)$. Figure 6 shows $J(\theta)$ and $J(0)\cos^2\theta$ plotted together, to give some sense of the correction due to integrating over all angles entering the detectors.

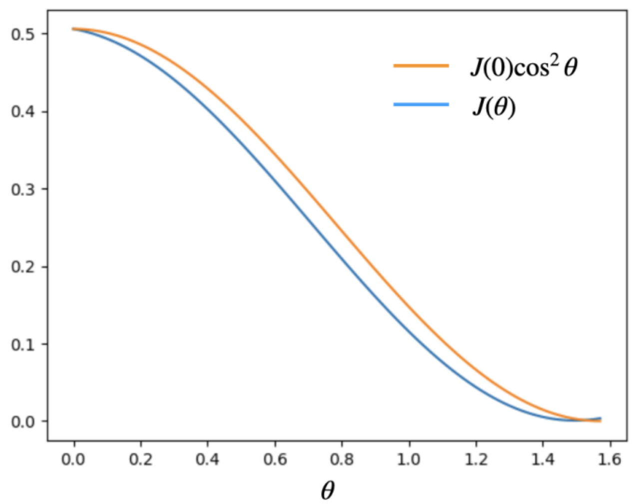

FIG. 6. Plot of $J(\theta)$ and $J(0)\cos^2\theta$.

IV. DATA AND ANALYSIS

As stated before, we ran the detectors at several angles for multi-hour runs. At a given angle θ, if we got N counts in time T, we report the observed rate As

$$R(\theta) = \frac{N}{T} \pm \frac{\sqrt{N}}{T}, \tag{2}$$

since the number of counts is a Poisson random variable, so the estimate on the mean is $N \pm \sqrt{N}$.

We measured the angle θ using iPhone software, which tells us the angle that the iPhone is with respect to the vertical. We verified some of the angles directly using a trigonometric calculation based on the geometry of the setup, and confirmed that the software was giving correct values within half a degree.

Shown below is a table of the raw data. Here the runtime T is reported in hours and minutes, and θ is reported in degrees.

$90° - \theta$	T	N
0	23:04	59
17	12:35	48
28	8:21	48
39	5:55	68
45	6:30	93
55	8:54	134
69	10:40	210
90	3:31	68

After converting this to a count rate using (2), and fitting to (1), we get values of

$$\alpha = [0.59 \pm 0.03] \frac{\text{counts}}{\text{min} \cdot \text{sr} \cdot \text{cm}^2}$$

and

$$J_b = [0.037 \pm 0.005] \frac{\text{counts}}{\text{min}}.$$

Figure 7 shows the linear fit between $R(\theta)$ and $J(\theta)$, along with the χ^2 value and χ^2 probability. The χ^2 probability of 0.80 suggests that this model is a decent fit.

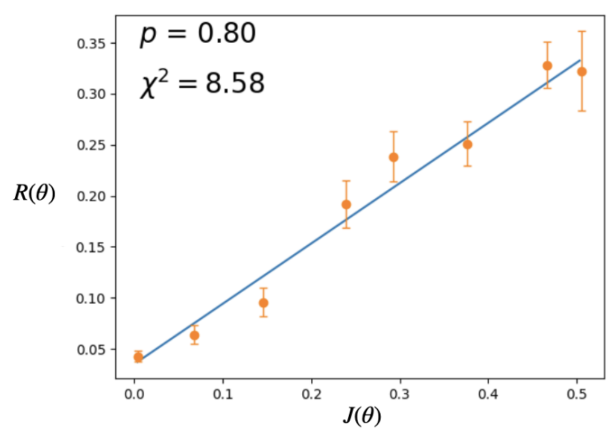

FIG. 7. Linear fit of $R(\theta)$ against $J(\theta)$.

V. SYSTEMATIC UNCERTAINTY ANALYSIS

V.1. Detector Sag

The first source of systematic uncertainty is a sag in the duct tape, which caused one of the detectors to shift its orientation by some fixed value, less than 2°. Figure 8 shows a picture of the sag. Due to the nature of the duct taping scheme, this sag angle was fairly consistent throughout all runs, at high and low angles.

FIG. 8. Slight sag in top detector due to duct tape coming loose.

Modeling the exact geometry of this sag is difficult (though doable), so we instead use a rough approximation to get a sense of the magnitude of the error this causes. Our approach is to treat the sag as a systematic shift in all of our θ values by some fixed $\Delta\theta \in (-2°, 2°)$. We recalculated α and β assuming these systematic shifts in θ, and estimated the systematic uncertainty as the width of this spread. Using this methodology, we have measured values

$$\alpha = [0.59 \pm 0.03^{\text{stat}} \pm 0.02^{\text{syst}}]\frac{\text{counts}}{\text{min} \cdot \text{sr} \cdot \text{cm}^2}$$

and

$$J_b = [0.037 \pm 0.005^{\text{stat}} \pm 0.003^{\text{syst}}]\frac{\text{counts}}{\text{min}}.$$

V.2. Accidental Coincidence Rate

The second source of systematic uncertainty is accidental coincidences, which form the base background rate of coincidences one expects given that the coincidence discriminator has some fixed time window τ.

In particular, if two events trigger within τ time of each other, then they are said to be coincident. Given count rates N_1 and N_2 for two detectors, we then expect an accidental coincidence rate of $2\tau N_1 N_2$ (see [3]). This contributes to a negative systematic correction to J_b, since we aren't interested in these accidental coincidences.

We aren't able to directly get a good estimate of τ for our setup, so we instead use the value $\tau \approx 30\,\mu$s given in [3] for the cosmic watch coincidence setup. With coincidence turned off, the detectors record $N \approx 2$ Hz, so we expect the accidental coincidence rate to be

$$J_{ac} \approx 240 \cdot 10^{-6}\ \text{Hz} \approx 0.01\frac{\text{counts}}{\text{min}}.$$

This is of similar order of magnitude as J_b, but is still significantly smaller, suggesting that there are other effects contributing to the background J_b.

VI. CONCLUSION

Our experimental results suggest that the $\cos^2\theta$ law for cosmic ray muon flux is a reasonable model, given the accuracy of the χ^2-fit to a theoretical model based on this law.

The background rate is partially explained by accidental coincidences, but there is still a remaining background after subtracting out the accidental coincidence rate. This could be explained due to cosmic ray showers, which are short events where the base count rate N becomes much higher, causing a much larger number of accidentals than expected for a short period of time.

ACKNOWLEDGEMENTS

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[1] *CosmicWatch: The Desktop Muon Detector (v3).* Axani S. and Conrad J. MIT Department of Physics (2020).

[2] *Measurement of Cosmic Muon angular distribution and vertical integrated flux by 2 m × 2 m RPC stack at IICHEP-Madurai.* Pethuraj et. al. JCAP (2017).

[3] *The Physics Behind the CosmicWatch Desktop Muon Detectors.* Axani S. MIT Department of Physics (2019).