Measuring Fine Structure Splitting in Sodium through Optical Spectroscopy

Gopal K. Goel (MIT)

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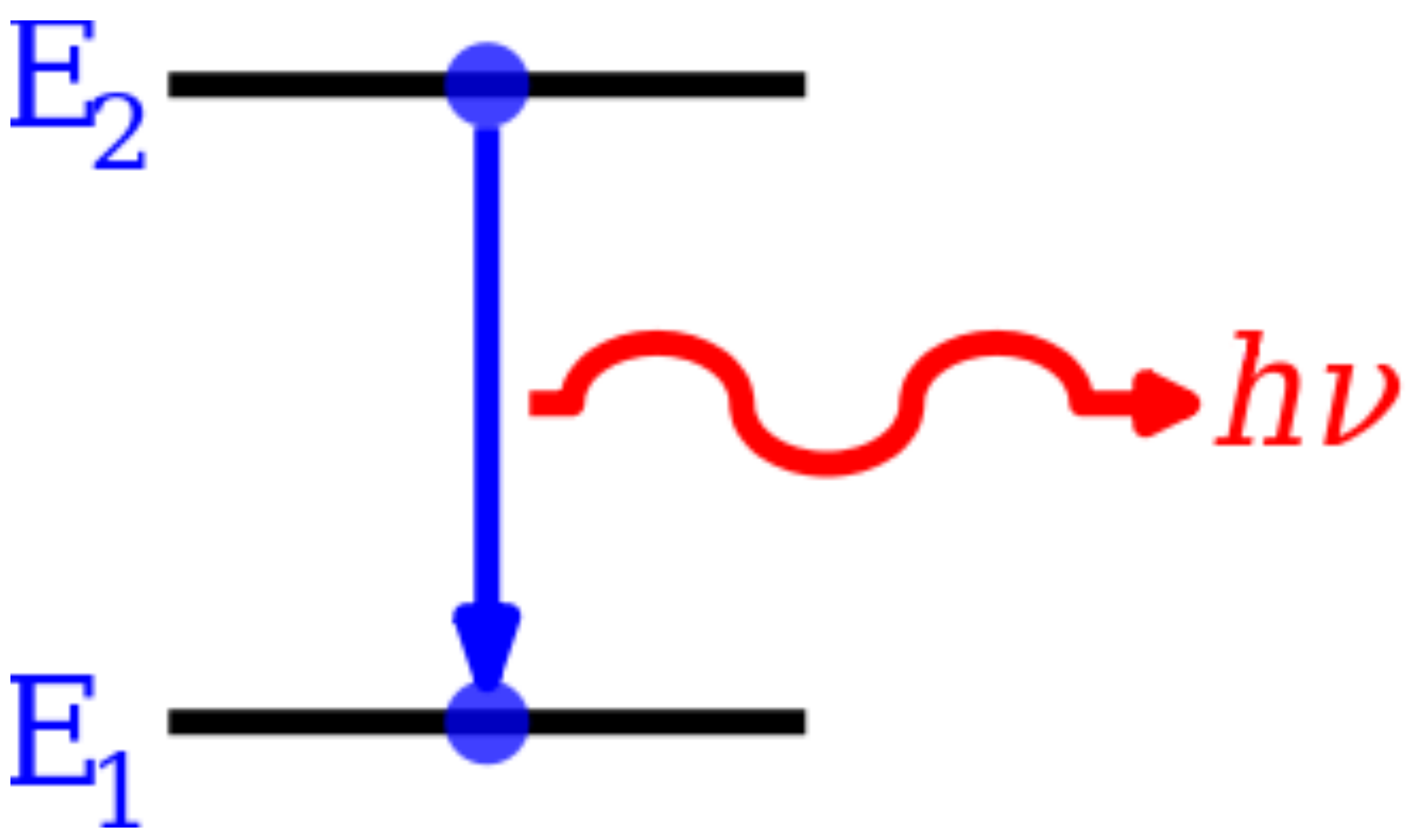
Emission lines of Atoms inform internal energy level structure

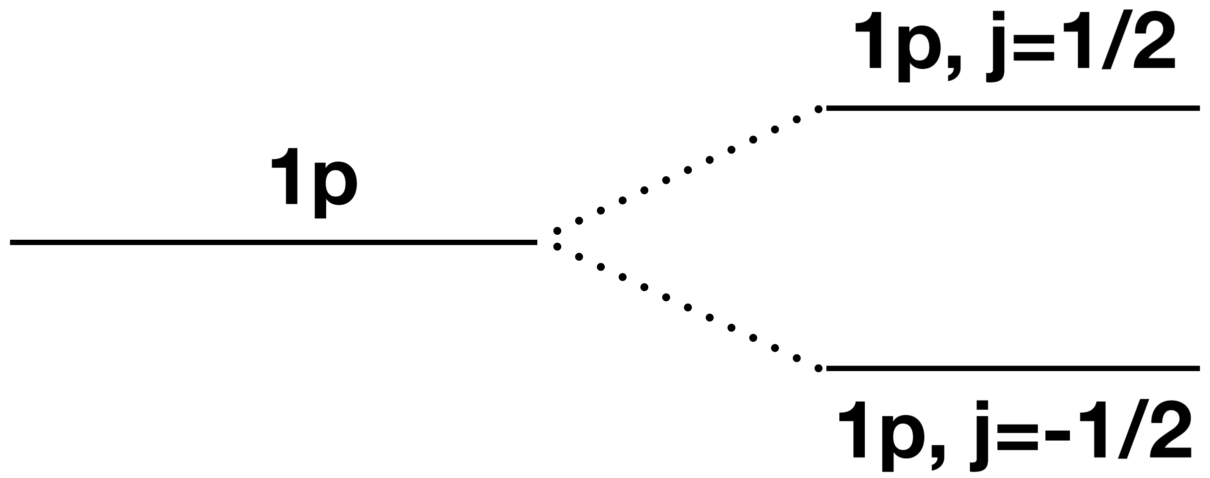
- We are very interested in the various quantum states of an atom
- If an electron transitions from a higher energy state to a lower energy state, it often emits a photon
- By measuring the energy of that photon, we can learn about the differences of energy levels in the atom, which tells us about the structure of the quantum states



Schematic diagram of atomic emission (https://en.wikipedia.org/wiki/Emission_spectrum)

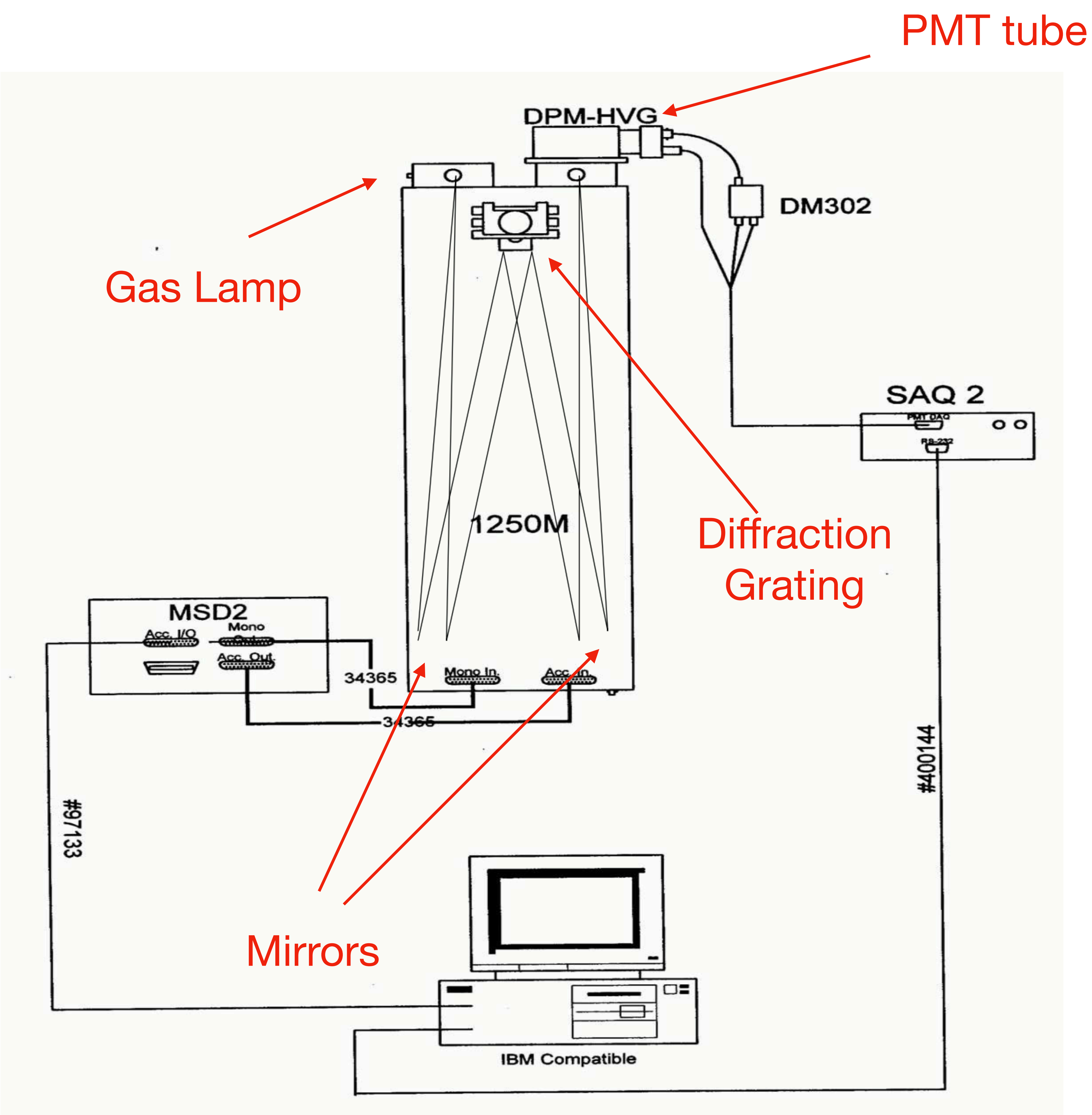
Goal: Observe evidence of fine structure

- Fine structure effects cause the energy levels of an atom to split
- Most accessible for sodium, given it is hydrogen-like atom, and its fine structure splits are much larger than that of hydrogen
- Goal is to measure these fine structure shifts for various transitions in sodium, and see if they match our theoretical understanding



Monochromator Setup

- Gas lamp emits light due to atomic emission of the gas
- Moving the diffraction grating tunes which band of wavelengths make it to the photomultiplier tube (PMT)
- Software reads out light output as a function of wavelength
- Wavelength is uncalibrated due to the imprecision of the internal counter system of the monochromator



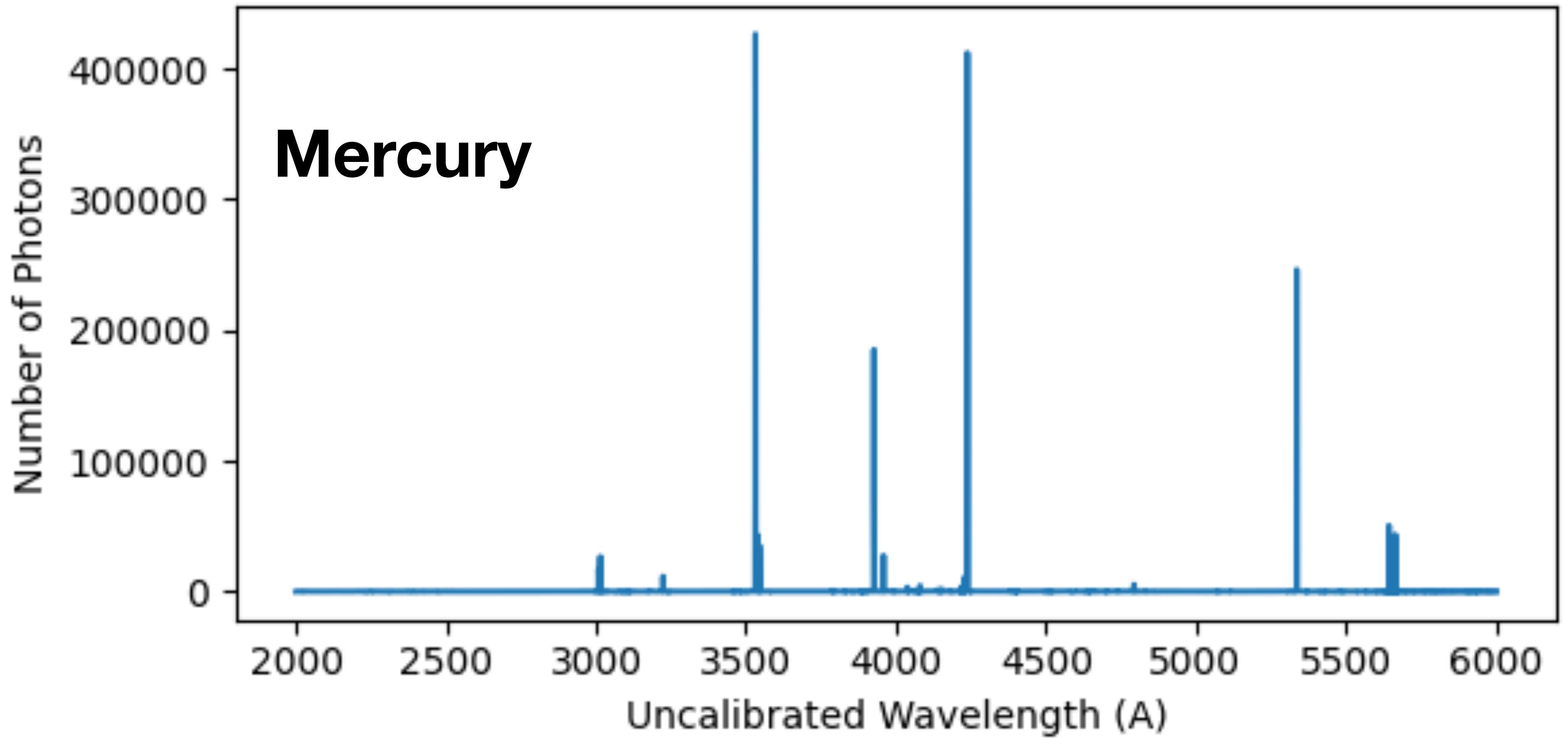
Experimental Setup

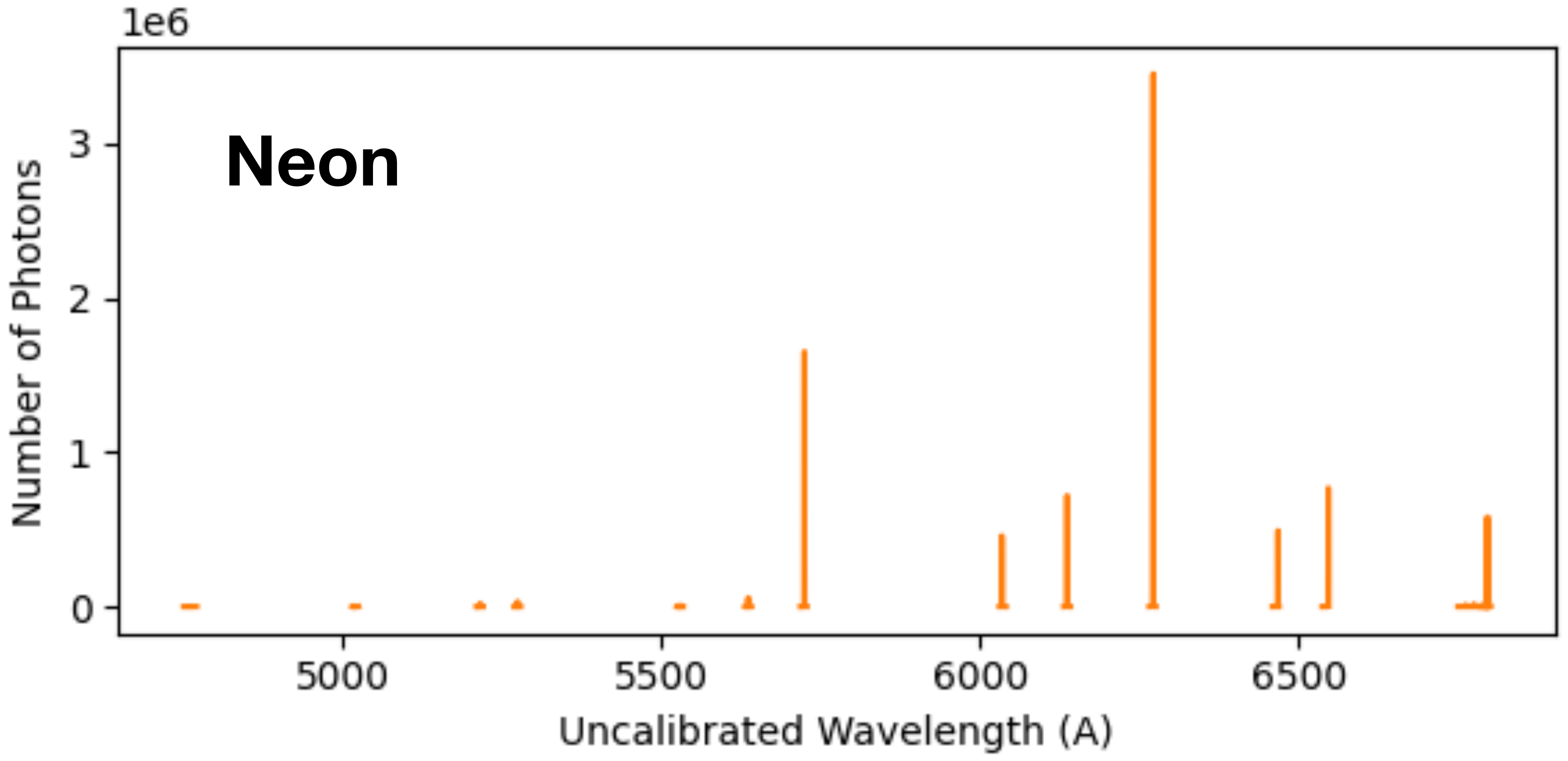
https://canvas.mit.edu/courses/19620/pages/exp17-optical-emission-spectra-of-hydrogenic-atoms?module_item_id=795283

Calibration Wavelengths

- Used known values of peaks of mercury and neon

- Mercury lines mostly in $[3000{,}6000]\ \mathrm{\AA}$ and neon lines mostly in $[5000{,}7000]\ \mathrm{\AA}$, so a wide range is covered





Calibration Procedure

- We perform a quadratic fit of uncalibrated wavelengths as a function of true wavelengths:

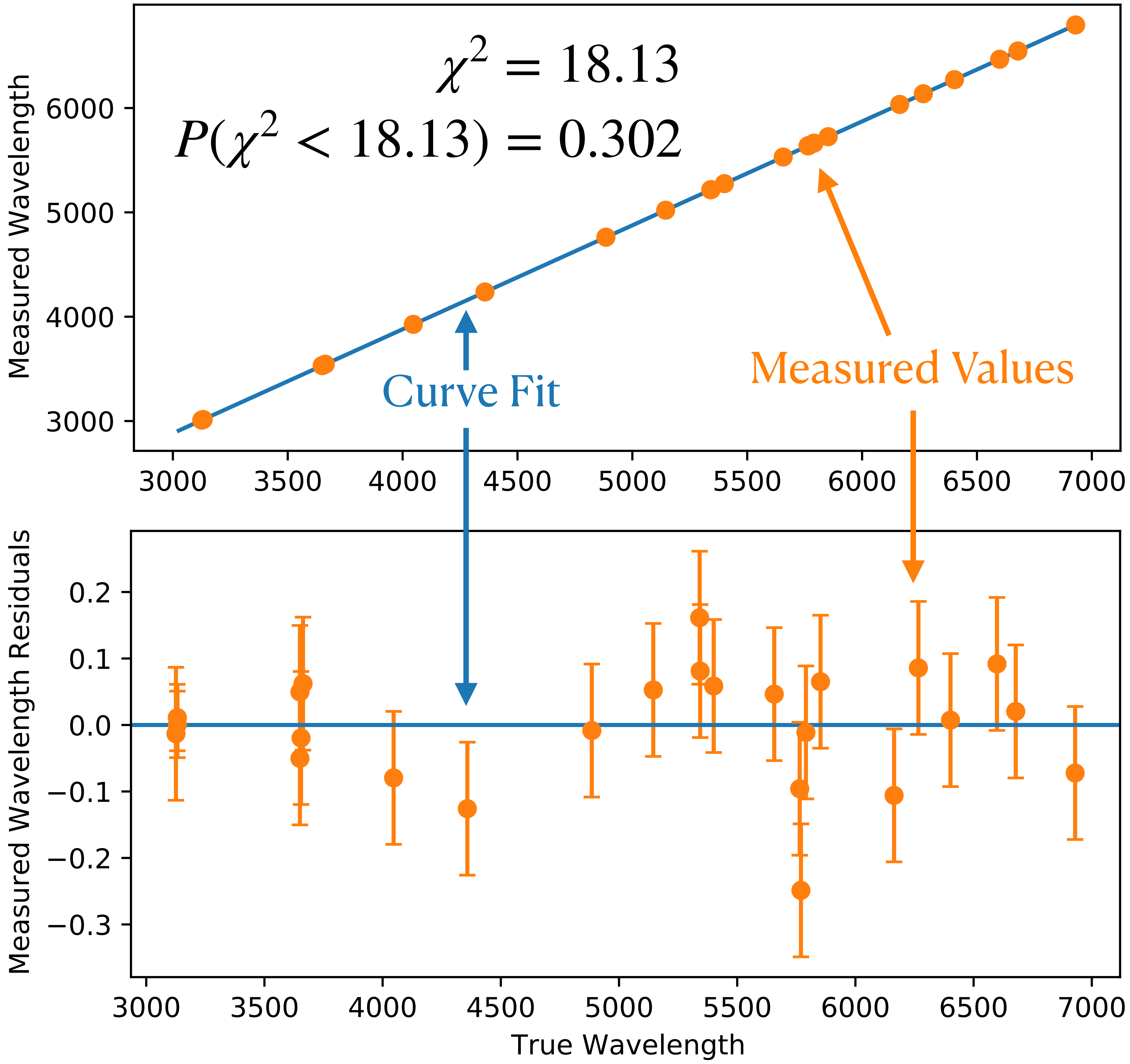
$$\lambda_{uc} = a\lambda_t^2 + b\lambda_t + c$$

- Fit values with uncertainties:

$$a = (-4.56 \pm 0.15) \cdot 10^{-7}\ \text{Å}^{-1}$$

$$b = 1.0004 \pm 0.0001$$

$$c = -114.1 \pm 0.3\ \text{Å}$$



$\chi^2 = 18.13$

$P(\chi^2 < 18.13) = 0.302$

Curve Fit

Measured Values

Estimation of Systematic Error from Calibration

- Main source of systematic error is from many scans affecting the calibration
- Comparing mercury scans separated by 5 days, we find that the uncalibrated wavelengths for the $3130\,\text{Å}$ doublet are off by less than $0.1\,\text{Å}$, so we can upper bound the systematic uncertainty in the true wavelength by $0.1\,\text{Å}$

Schrödinger Theory of Hydrogen

- Solve Schrödinger's equation with $V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}$.

- States are parameterized by quantum numbers (n, ℓ, m, s) where $n = 1,2,3,\ldots, \ell = 0,1,\ldots, n-1, m = -\ell, \ldots, \ell$, and $s = \pm\frac{1}{2}$.

- Useful later to use a basis with coupled angular momentum $J = L + S$, where states have quantum numbers (n, ℓ, j, m_j) with $j = \ell \pm \frac{1}{2}$ and $m_j = -j, \ldots, j$.

Spin-Orbit Coupling

- Electron has spin magnetic moment μ
- Electron experiences a magnetic field due to relative motion with proton
- Hamiltonian needs correction term $\delta H = -\mu \cdot B$
- The correction term is given exactly by

$$\delta H = \frac{1}{2mc^2}\frac{1}{r}\frac{\partial V(r)}{\partial r} L \cdot S$$

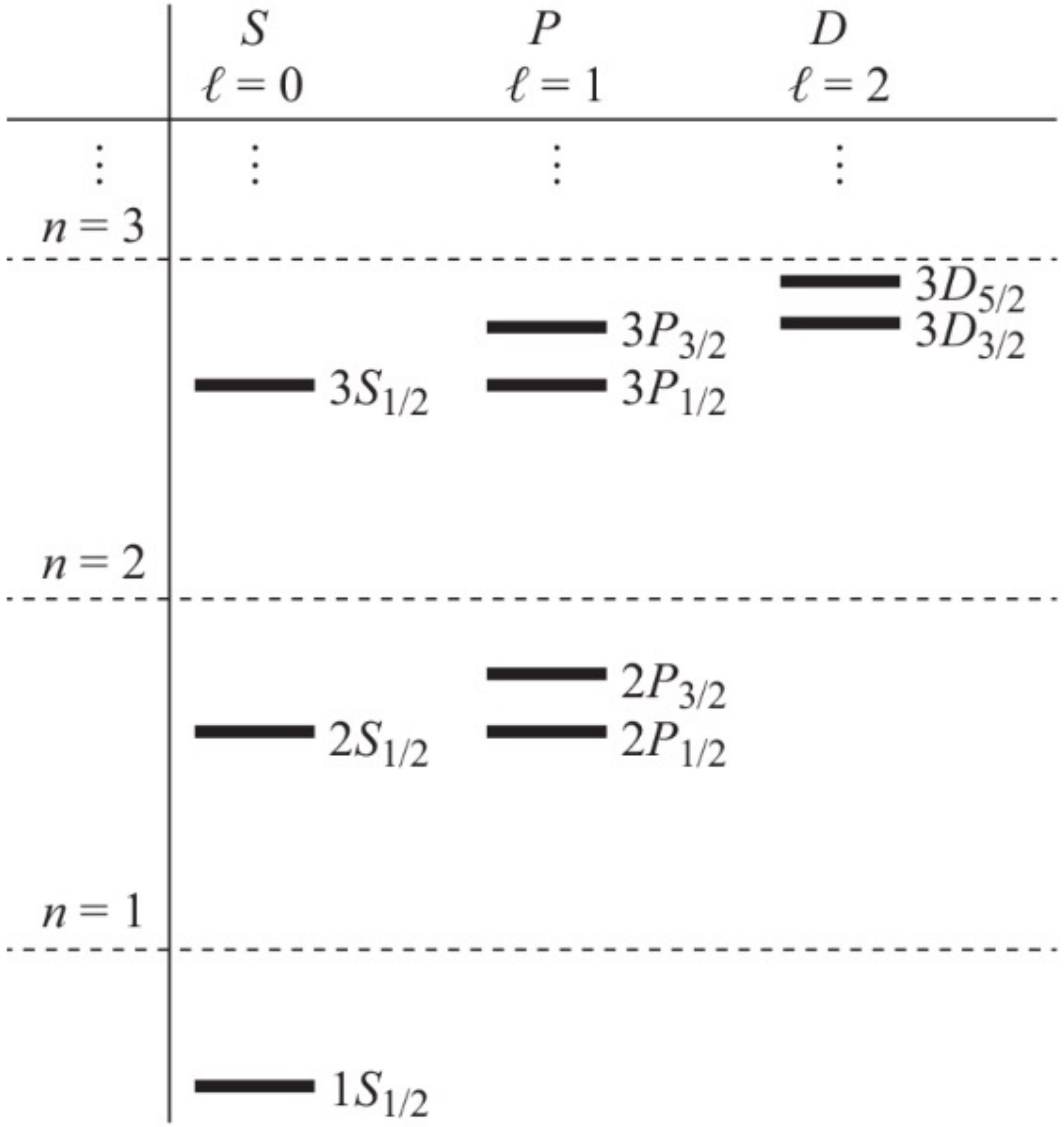
Fine structure splitting

- Perturbation δH is diagonal in coupled basis (n, ℓ, j, m_j)
- States with (n, ℓ) get energy levels split based on value of j
- Splitting between $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$ states is given by

$$\frac{e^2\alpha^2}{8\pi\varepsilon_0}\frac{1}{n^3\ell(\ell+1)}$$

Hydrogen energy levels

- Refer to $\ell = 0,1,2$ states as S, P, D respectively
- No splitting for S states



Energy level diagram for hydrogen (not to scale) (From *Mastering Quantum Mechanics* by Barton Zwiebach)

Fine structure splitting in Sodium similar but no longer exact

- Story is the same, except that the formula for the splitting is not exact anymore
- The $n = 1$ and $n = 2$ shells are filled up with electrons, so the ground state is 3s
- Observe emission peaks in doublet pairs due to fine structure splitting

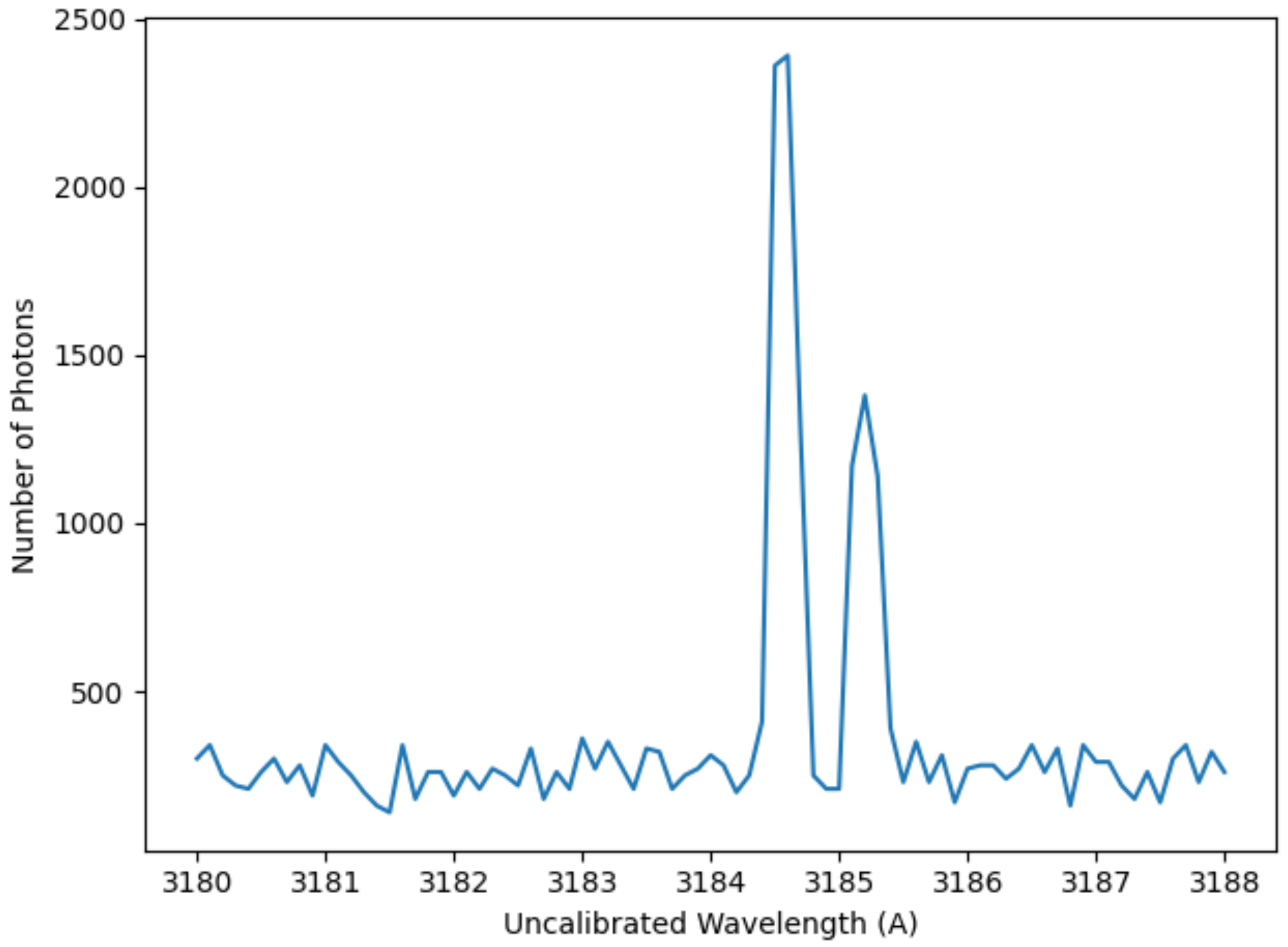
Observed sodium doublets

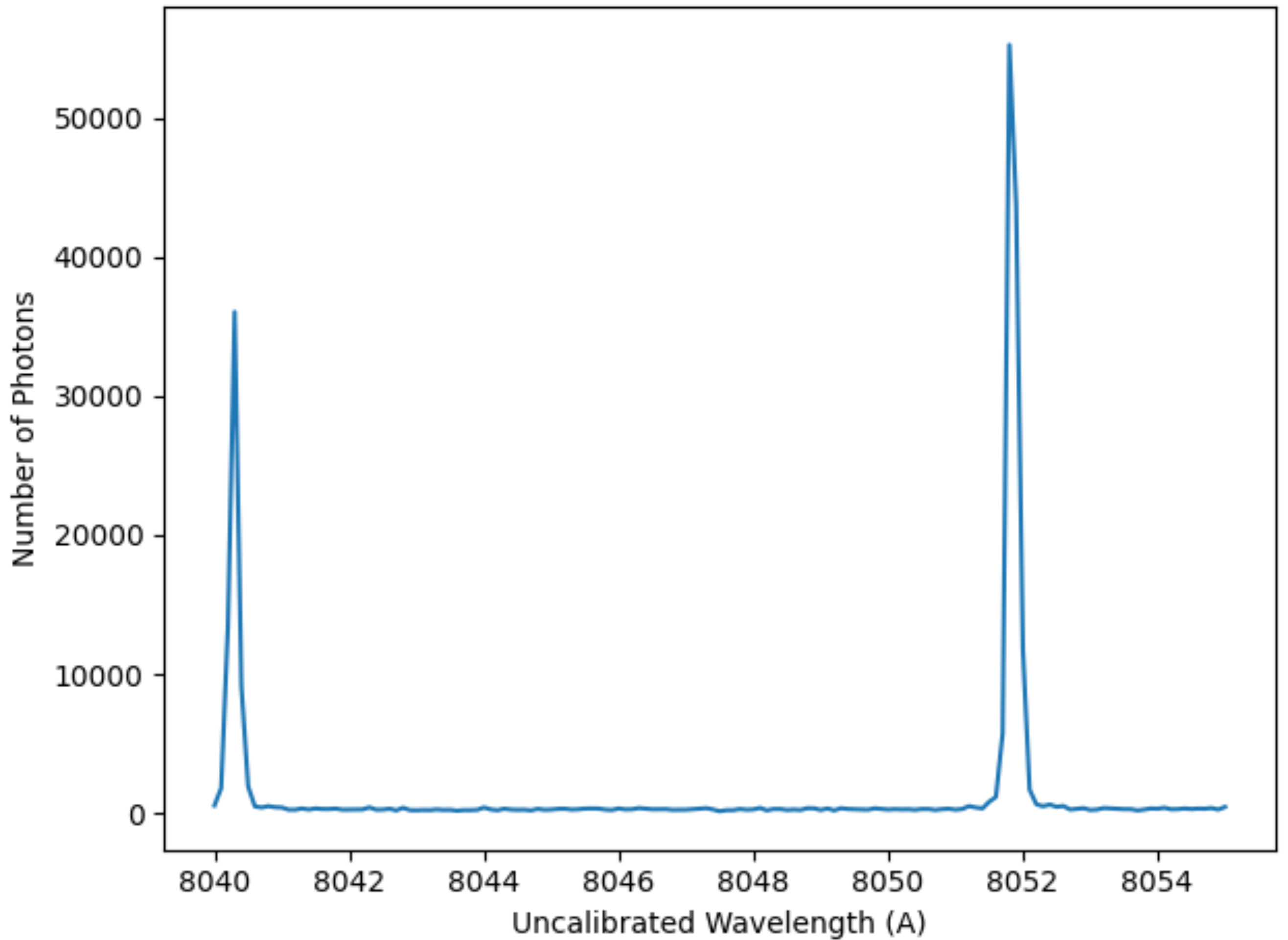
We measured three doublet lines. Using calibration function and Monte-Carlo error estimates using the calibration parameter uncertainty, we find the three pairs to be:

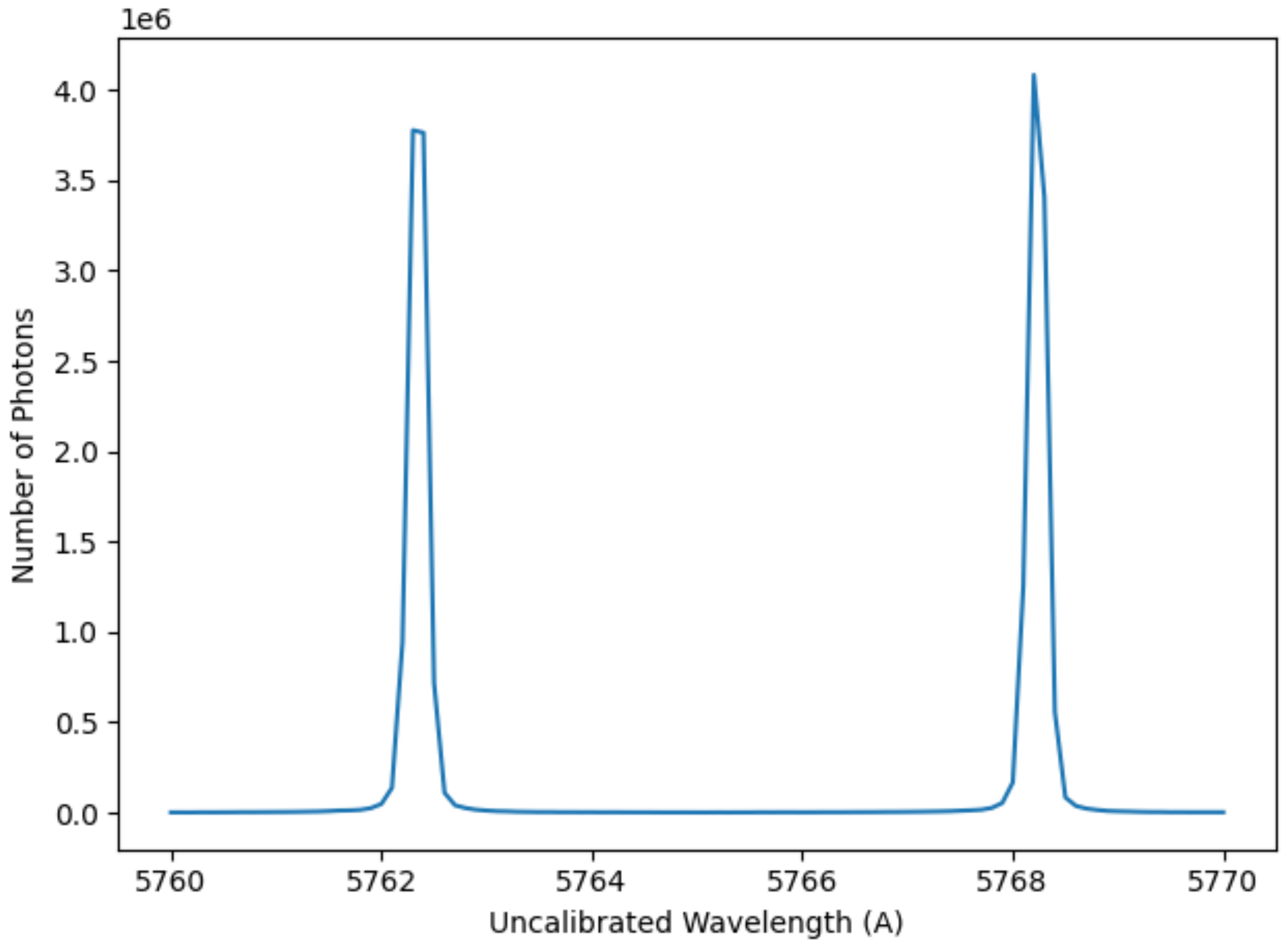
$[3302.3 \pm 0.6]$ Å, $[3302.9 \pm 0.6]$ Å

$[5889.9 \pm 1.1]$ Å, $[5895.8 \pm 1.1]$ Å

$[8181.6 \pm 1.6]$ Å, $[8193.4 \pm 1.6]$ Å







Identifying State Transitions

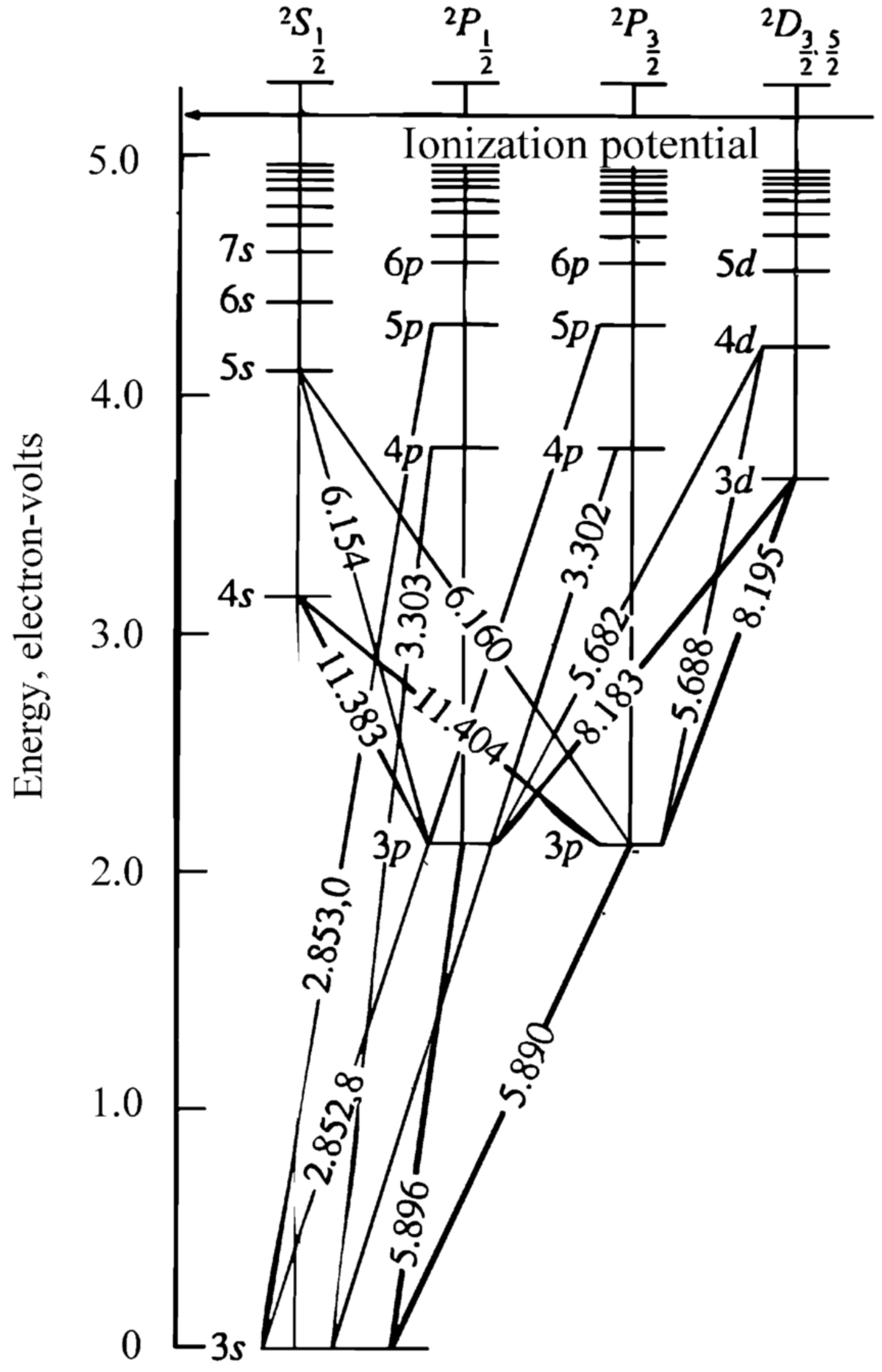
Using the known energy levels of sodium (without doublet splitting), we identify the pairs as:

4p→3s: [3302.3 ± 0.6] Å, [3302.9 ± 0.6] Å

3p→3s: [5889.9 ± 1.1] Å, [5895.8 ± 1.1] Å

3d→3p: [8181.6 ± 1.6] Å, [8193.4 ± 1.6] Å

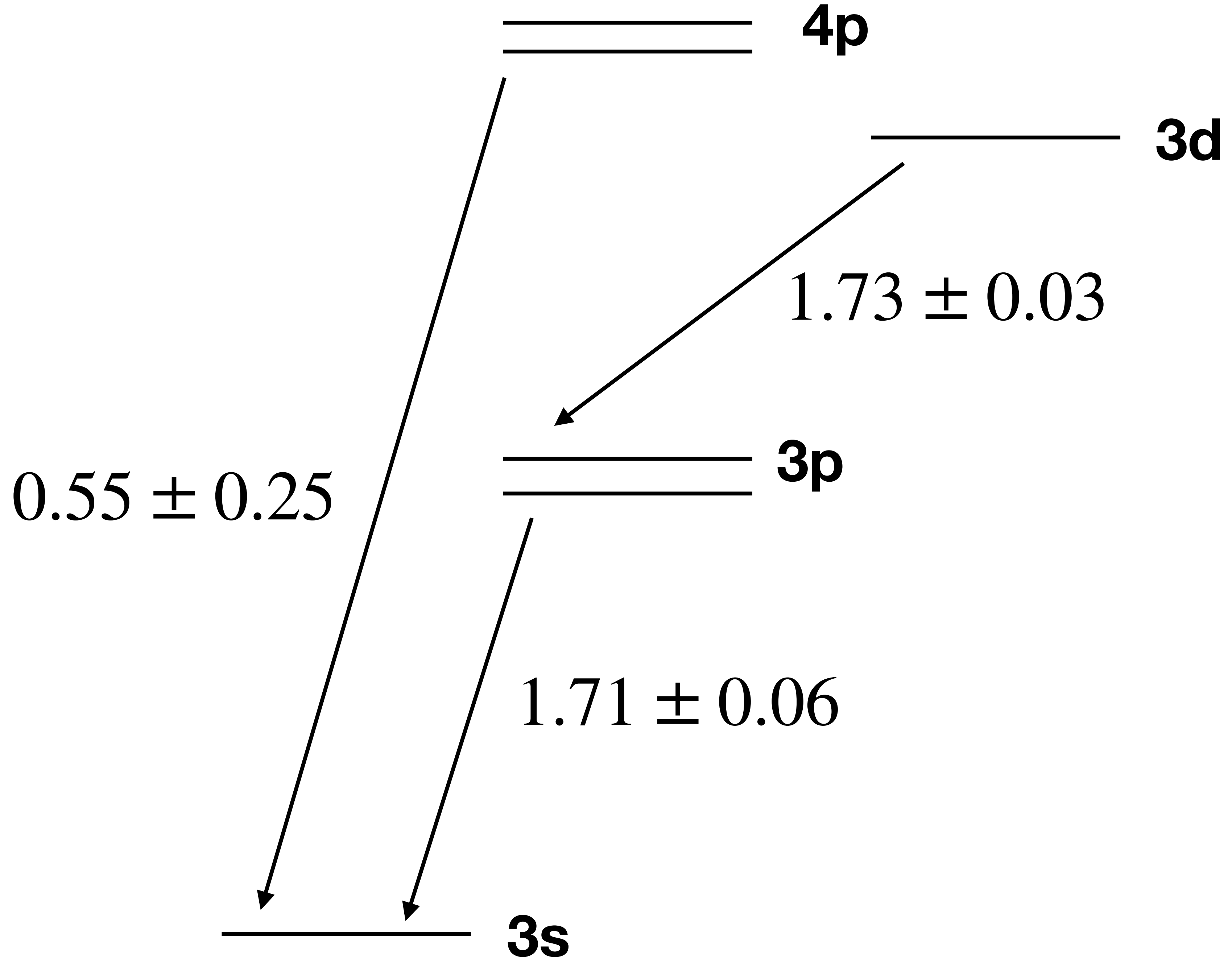
All these errors are highly correlated, so we later Monte-Carlo estimate the error on the difference



Energy levels of sodium with transition wavelengths
(A General Overview of Atomic Spectrometric Techniques by Pereiro)

Analyzing doublet separations

- Level 3s has no splitting, and 3d splitting is negligible compared to p splitting
- Wavenumber (inverse wavelength) is proportional to energy
- Used Monte-Carlo error estimation on the wavenumber difference
- As expected, doublet separation same for 3p transitions, within statistical error
- Ratio of 4p and 3p separation is 0.3 ± 0.1. Using the $\frac{1}{n^3}$ heuristic, the ratio should be around 0.42



Measured transitions labeled with doublet wavenumber separation in 10^{-7}Å^{-1}

Conclusion

- Observed the doublet structure in sodium emission lines
- Measured the doublet wavenumber separation and matched it to theoretical heuristics for the fine structure splitting